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Hierarchical matrices in the see-saw mechanism, large neutrino mixing and leptogenesis

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Abstract. We consider the see-saw mechanism for hierarchical Dirac and Majorana neutrino mass matrices $m_{\rm D}$ and $M_{\rm R}$, including the CP violating phases. Simple arguments about the structure of the neutrino mass matrix and the requirement of successful leptogenesis lead to the situation that one of the right-handed Majorana neutrinos is much heavier than the other two, which in turn display a rather mild hierarchy. It is investigated how for the neutrino mixing one small and two large mixing angles are generated. The mixing matrix element $|U_{e3}|^2$ is larger than 10^{-3} and a characteristic ratio between the branching ratios of lepton flavor violating charged lepton decays $\ell_j \to \ell_i \gamma$ is found. Successful leptogenesis implies sizable CP violation in oscillation experiments. As in the original minimal see-saw model, the signs of the baryon asymmetry of the universe and of the CP asymmetry in neutrino oscillations are equal and there is no connection between the leptogenesis phase and the effective mass as measurable in neutrinoless double beta decay.

1 Introduction

The fact that two mixing angles in the neutrino mixing matrix are large [1] is a severe difference with respect to the quark sector. In the latter, hierarchical mass matrices are the most natural explanation for small mixing angles. Thus, it is natural to assume that in a GUT framework also the Dirac mass matrix $m_{\rm D}$ and the Majorana mass matrix $M_{\rm R}$, both appearing in the see-saw mechanism [2], are of hierarchical structure, i.e., of close to diagonal form. In the see-saw mechanism the neutrino mass matrix m_ν is a matrix product containing $m_{\rm D}$ and $M_{\rm R}$. Consequently, it is possible that m_ν does not display a close to diagonal structure, in contrast to the fundamental matrices $m_{\rm D}$ and $M_{\rm R}$ [3]. Accordingly, the observed neutrino mixing can take the characteristic form with two large angles and one small one. The purpose of the present note is to readdress this point including effects of the CP phases and investigate its consequences for leptogenesis and for the branching ratios of lepton flavor violating (LFV) charged lepton decays like $\mu \to e\gamma$. In order to reach a hierarchical mass spectrum, the 23 block of m_ν has to be approximately degenerate with entries larger than the remaining elements [4, 5]. Working within useful parameterizations of $m_{\rm D}$ and $M_{\rm R}$, these requirements lead to the possibility that one of the right-handed Majorana neutrinos is much heavier than the other two. Successful leptogenesis then implies a rather mild hierarchy between the latter. In this simple framework one can obtain neutrino mixing phenomenology in accordance with the data; one predicts $|U_{e3}|^2 \gtrsim 10^{-3}$ and finds a characteristic ratio of the branching ratios of the LFV charged lepton decays. The baryon asymmetry of the universe and the CP asymmetry measurable in neutrino oscillations are directly connected.

In Sect. 2 we will briefly review the formalism of neutrino mixing and leptogenesis. We investigate how hierarchical Dirac and Majorana mass matrices lead to large neutrino mixing in a simplified 2×2 case in Sect. 3. The realistic 3×3 case is treated in Sect. 4, where also the predictions for leptogenesis and low energy observables are investigated. We conclude in Sect. 5.

2 Framework

The neutrino mass matrix is given by the see-saw mechanism [2] as

$$m_\nu \simeq -m_{\rm D} M_{\rm R}^{-1} m_{\rm D}^{\rm T} \, , \qquad (1)$$

where $m_{\rm D}$ is a Dirac mass matrix and $M_{\rm R}$ the mass matrix of the right-handed Majorana neutrinos. We shall work in a basis in which both the charged lepton mass matrix and $M_{\rm R}$ are real and diagonal, i.e., $M_{\rm R} = {\rm diag}(M_1, M_2, M_3)$ with real $M_3 > M_2 > M_1$. The largest mass M_3 is expected to lie around or below the unification scale $M_{\rm GUT} \simeq 10^{16}$ GeV. The matrix m_ν is observable in terms of

$$m_\nu = U^\dagger m_\nu^{\rm diag} U^* \, . \qquad (2)$$

Here $m_\nu^{\rm diag}$ is a diagonal matrix containing the light neutrino mass eigenstates m_i, and U is the unitary Pontecorvo–

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Maki–Nagakawa–Sakata [6] lepton mixing matrix, which can be parametrized as

$$U = O_{23} O_{13}^{\delta} O_{12} P \,, \tag{3}$$

where O_{ij} is a rotation matrix. E.g.,

$$O_{13}^{\delta} = \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix} , \tag{4}$$

where $c_{13} = \cos\theta_{13}$ and $s_{13} = \sin\theta_{13}$ and δ is the "Dirac phase" measurable in neutrino oscillations. The matrices O_{12} and O_{23} are real and P is a diagonal phase matrix containing the two additional Majorana phases. In total,

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} e^{i\delta} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} e^{i\delta} \end{pmatrix} \times \operatorname{diag}\left(1, e^{i\alpha}, e^{i\beta}\right) . \tag{5}$$

The observation in previous experiments [1] as well as inclusion of the recent SNO salt phase data [7] implies the following values of the oscillation parameters [8], given at 3σ:

$$\begin{aligned} \tan 2\theta_{12} &\simeq 1.5 \ldots 4.4 \,, \\ \tan 2\theta_{13} &\lesssim 0.45 \,, \\ |\tan 2\theta_{23}| &\gtrsim 2 \,, \\ \Delta m^2_{\odot} &\simeq (5.4 \ldots 9.5) \cdot 10^{-5} \mathrm{eV}^2 \,, \\ \Delta m^2_{\mathrm{A}} &\simeq (1.4 \ldots 3.7) \cdot 10^{-3} \mathrm{eV}^2 \,. \end{aligned} \tag{6}$$

Typical best-fit points are $\tan^2\theta_{12} = 0.45$ and $\theta_{23} = \pi/4$, corresponding to $\tan 2\theta_{12} \simeq 2.4$ and $\tan 2\theta_{23} \gg 1$. We have therefore two large and one small mixing angle, in sharp contrast to the situation present in quark mixing.

The presence of heavy right-handed Majorana neutrinos in the see-saw mechanism means that the possibility of leptogenesis [9] is included. Thus, the see-saw mechanism gains a large amount of attractiveness. Leptogenesis explains the baryon asymmetry of the universe through the CP asymmetric out-of-equilibrium decay of heavy right-handed Majorana neutrinos occurring much before the electroweak phase transitions. It is governed by the decay asymmetry [9,10]

$$\varepsilon_1 \simeq \frac{1}{8\pi v^2} \frac{1}{\left(m_{\mathrm{D}}^{\dagger} m_{\mathrm{D}}\right)_{11}} \sum_{j \neq i} \mathrm{Im} \left(m_{\mathrm{D}}^{\dagger} m_{\mathrm{D}}\right)^2_{1j} f\left(M_j^2/M_1^2\right) , \tag{7}$$

where $f(x)$ is a function whose limit for $x \gg 1$, i.e., hierarchical neutrinos, is $-3/\sqrt{x}$. The necessary values of $|\varepsilon_1| \gtrsim 10^{-7}$ and $M_1 \gtrsim 10^9\,\mathrm{GeV}$ are expected in order to produce a sufficient baryon asymmetry [10,11]. There is a tendency of this lower mass limit to be in conflict with bounds on the reheating temperature, which stem from the requirement that the decay products of the gravitino do not spoil Big Bang Nucleosynthesis predictions. From this condition one finds upper limits of less than $M_1 \lesssim 10^9 \ldots 10^{10}$ GeV [12]. The baryon asymmetry is positive when ε_1 is negative, because we have $Y_B \propto c\varepsilon_1$ [10], where c is a negative constant stemming from the conversion of the lepton asymmetry into a baryon asymmetry.

3 The 2 × 2 case

We shall analyze the generation of large mixing in m_ν from hierarchical m_{D} and M_{R} first in a simplified 2×2 framework. Consider a complex symmetric matrix

$$m = \begin{pmatrix} a & b \\ b & d \end{pmatrix} , \tag{8}$$

which is diagonalized by a unitary matrix U through

$$m^{\mathrm{diag}} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} = U^{\mathrm{T}} m U \,,$$

where

$$U = \begin{pmatrix} \cos\theta & \sin\theta e^{i\phi} \\ -\sin\theta e^{-i\phi} & \cos\theta \end{pmatrix} . \tag{9}$$

In general, a symmetric matrix 2×2 is diagonalized by UP, where U is given above and P is a diagonal phase matrix. By redefining the charged lepton fields, these two additional phases can be absorbed. The eigenvalues m_1 and m_2 with $m_2 > m_1$ are trivial to obtain. The mixing angle θ is given by the equation

$$\tan 2\theta = \frac{2b}{d e^{-i\phi} - a e^{i\phi}} \,. \tag{10}$$

The phase ϕ is defined by the requirement of the angle θ being real, i.e.,

$$\arg(b) = \arg\left(d e^{-i\phi} - a e^{i\phi}\right) . \tag{11}$$

Now consider in a simple 2×2 case hierarchical Dirac and Majorana mass matrices, i.e.,

$$m_{\mathrm{D}} = m \begin{pmatrix} \epsilon_{\mathrm{D}}^2 & A\epsilon_{\mathrm{D}} \\ B\epsilon_{\mathrm{D}} & 1 \end{pmatrix} \quad \text{and} \quad M_{\mathrm{R}} = M \begin{pmatrix} \epsilon_M & 0 \\ 0 & 1 \end{pmatrix} , \tag{12}$$

with $\epsilon_{\mathrm{D}}, \epsilon_M \ll 1$ but an unspecified hierarchy between ϵ_{D} and ϵ_M. The complex coefficients $A = a e^{i\alpha}$ and $B = b e^{i\beta}$ with real a and b have absolute values of order 1. Inserting the matrices in the see-saw formula (1) yields

$$m_\nu = -\frac{m^2}{M} \begin{pmatrix} \frac{\epsilon_{\mathrm{D}}^4}{\epsilon_M} + A^2 \epsilon_{\mathrm{D}}^2 & A\epsilon_{\mathrm{D}} + B\frac{\epsilon_{\mathrm{D}}^3}{\epsilon_M} \\ \cdot & 1 + B^2 \frac{\epsilon_{\mathrm{D}}^2}{\epsilon_M} \end{pmatrix}$$

$$= -\frac{m^2}{M} \begin{pmatrix} \epsilon_{\rm D}^2 \left(A^2 + \eta\right) & \epsilon_{\rm D}(A + B\eta) \\ \cdot & 1 + B^2\eta^2 \end{pmatrix}, \qquad (13)$$

where we defined the characteristic quantity $\eta \equiv \epsilon_{\rm D}^2/\epsilon_M$. The magnitude of the mixing angle is therefore governed by the ratio of the hierarchies of the Dirac and Majorana masses. Namely,

$$\tan 2\theta = 2\epsilon_{\rm D} \frac{A + \eta B}{1 + \eta \left(B^2 - e^{2i\phi} A^2 \epsilon_M - e^{2i\phi}\epsilon_{\rm D}^2\right)}\, e^{i\phi}. \qquad (14)$$

From (14) one encounters several interesting special cases, some of which are discussed in the following.

(1) $\eta \simeq 1$ but $\epsilon_{M,D} \ll 1$: similar hierarchy in $m_{\rm D}$ and $M_{\rm R}$.

Now, we find for the mass matrix and the mixing angle

$$m_\nu \simeq -\frac{m^2}{M} \begin{pmatrix} 0 & \epsilon_{\rm D}(A + B) \\ \cdot & 1 + B^2 \end{pmatrix}$$

$$\Rightarrow \tan 2\theta \simeq 2\epsilon_{\rm D} \sqrt{\frac{a^2 + b^2 + 2abc_{\alpha-\beta}}{1 + b^4 + 2b^2 c_{2\beta}}}. \qquad (15)$$

Values of $\beta \simeq \pi/2$ and $b \simeq 1$ can thus lead to (close to) maximal mixing as observed in the atmospheric neutrino oscillation experiments. In this case, $\phi \simeq -\arg(A{+}i)$. Also, relaxing the conditions for b and β a bit can lead to the observed large but not maximal mixing in solar neutrino oscillation experiments.

(2) $\eta \ll 1$: stronger hierarchy in $m_{\rm D}$.

The mass matrix and mixing are now given by

$$m_\nu \simeq -\frac{m^2}{M} \begin{pmatrix} 0 & A\epsilon_{\rm D} \\ \cdot & 1 \end{pmatrix} \Rightarrow \tan 2\theta \simeq 2\epsilon_{\rm D} a, \qquad (16)$$

which, for the large but still reasonable choices of $\epsilon_{\rm D} \simeq \sin\theta_C \simeq 0.22$ and $a \gtrsim 4$ yields $\tan 2\theta \gtrsim \sqrt{3}$, i.e., $\theta \gtrsim \pi/6$, as implied by the observed non-maximal large mixing in the solar neutrino oscillation experiments. Smaller values of $\epsilon_{\rm D}$ and a can easily reproduce the small mixing parameter as implied from the CHOOZ and Palo Verde reactor neutrino oscillation experiments. For the phase we have $\phi \simeq -\alpha$.

(3) $\eta \gg 1$: stronger hierarchy in $M_{\rm R}$.

The mixing is found to be

$$m_\nu \simeq -\frac{m^2}{M} \begin{pmatrix} 0 & B\epsilon_{\rm D}\eta \\ \cdot & B^2\eta \end{pmatrix} \Rightarrow \tan 2\theta \simeq 2\epsilon_{\rm D} \frac{1}{b}, \qquad (17)$$

for which similar arguments as for the case $\eta \ll 1$ hold. The phase is given by $\phi \simeq \beta$.

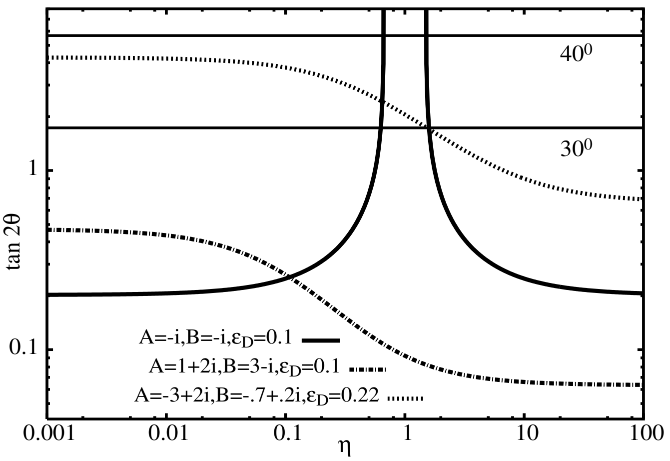

Fig. 1. Result for the mixing angle in a 2×2 framework, (14), obtained for hierarchical Dirac and Majorana neutrino mass matrices $m_{\rm D}$ and $M_{\rm R}$ and different values of the relevant parameters

To sum up, hierarchical Dirac and Majorana mass matrices reproduce for specific choices of the hierarchies and parameters all observed types of neutrino mixing: (close to) maximal, non-maximal large and small mixing. Exactly maximal and vanishing mixing requires some fine-tuning. Vanishing mixing would be obtained for $|A + \eta B| \simeq 0$ or equivalently $a^2 + b^2\eta^2 = -2abc_{\alpha-\beta}$. We show in Fig. 1 several examples of the mixing obtained with specific choices of $\epsilon_{\rm D}$, A and B. One finds from the figure and the discussion in this section that in order to obtain (close to) maximal mixing there is – in the given parametrization – a crucial dependence on the hierarchies of the fundamental matrices $m_{\rm D}$ and $M_{\rm R}$. Also the phases play an important role. Leptogenesis in turn requires the presence of CP violation[1] and – from (7) – depends on $m_{\rm D}$ and $M_{\rm R}$, and therefore also on the ratio of the hierarchies. We should thus analyze leptogenesis in this scenario. The decay asymmetry reads

$$\begin{aligned} \varepsilon_1 &= \frac{3\epsilon_M}{4\pi} \frac{m^2}{v^2} \frac{1}{b^2 + \epsilon_{\rm D}^2} \\ &\quad \times \left(\left(a\epsilon_{\rm D}^2 \cos\alpha + b\cos\beta\right) \left(b\sin\beta - a\epsilon_{\rm D}^2 \sin\alpha\right) \right) \\ &\simeq \frac{3\epsilon_M}{8\pi} \sin 2\beta\,, \end{aligned} \qquad (18)$$

where terms of order $\epsilon_{\rm D}^2$ were neglected and $m \simeq v$ was used. We can construct a very interesting special case: suppose that the mass matrix parameters take the values $b \simeq 1$, $\epsilon_{\rm D} \simeq 0.1$ and $\eta \simeq 1$. Then, from (15), we see that maximal mixing is only possible for $\beta \simeq \pi/2$. For this value of the phase, however, the decay asymmetry is highly suppressed. We see that maximal mixing implies a too small baryon asymmetry, or, in other words: requiring a non-zero baryon asymmetry implies non-maximal neutrino mixing. We shall encounter a slightly similar effect in the next section for the 3×3 case.

4 The 3 × 3 case

Let us turn now to the appropriate three flavor case. We can parameterize the relevant mass matrices $m_{\rm D}$ and $M_{\rm R}$

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[1] Note though that *in general* no link between low and high energy CP violation exists [13, 14] and any such connection will be model dependent

now as

$$m_{\rm D} \simeq m \begin{pmatrix} 0 & A\epsilon_{\rm D}^3 & 0 \\ B\epsilon_{\rm D}^3 & \epsilon_{\rm D}^2 & F\epsilon_{\rm D}^2 \\ 0 & g\epsilon_{\rm D}^2 & 1 \end{pmatrix}, \quad M_{\rm R} = M \begin{pmatrix} \epsilon_{M1} & 0 & 0 \\ 0 & \epsilon_{M2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{19}$$

For later use we define $A = a\mathrm{e}^{\mathrm{i}\alpha}$, $B = b\mathrm{e}^{\mathrm{i}\beta}$ and $F = f\mathrm{e}^{\mathrm{i}\phi}$; g can be chosen real. Again, the complex coefficients have absolute values of order 1, so one has g. Small entries in the 11, 13 and 31 elements of $m_{\rm D}$ are neglected (see below) and we have $\epsilon_{M1} < \epsilon_{M2}$. We choose now the following parameters describing the relative hierarchy in $m_{\rm D}$ and $M_{\rm R}$:

$$\eta_1 = \epsilon_{\rm D}^4/\epsilon_{M1} \text{ and } \eta_2 = \epsilon_{\rm D}^4/\epsilon_{M2} \text{ with } \eta_1 > \eta_2 . \tag{20}$$

The typical expansion parameter in $m_{\rm D}$ is $\epsilon_{\rm D} \simeq 0.1$, the overall mass scale $m \simeq v \simeq 174\,\mathrm{GeV}$. Using the see-saw formula we find for m_ν

$$m_\nu \simeq \frac{-m^2}{M} \begin{pmatrix} A^2\epsilon_{\rm D}^2\eta_2 & A\epsilon_{\rm D}\eta_2 & Ag\epsilon_{\rm D}\eta_2 \\ \cdot & \eta_2 + B^2\epsilon_{\rm D}^2\eta_1 + F^2\epsilon_{\rm D}^4 & F\epsilon_{\rm D}^2 + g\eta_2 \\ \cdot & \cdot & 1 + g^2\eta_2 \end{pmatrix}. \tag{21}$$

The light neutrino mass scheme will of course be hierarchical. To have an approximately degenerate spectrum in the 23 submatrix of m_ν (with scale $\sim \sqrt{\Delta m_{\rm A}^2}$) it is required that $g \simeq 1$ and $\eta_2 \simeq 1$ or $\eta_2 \simeq 10$. Larger values are incompatible with $m \simeq v$ and $M \lesssim 10^{16}\,\mathrm{GeV}$. Later on it will be shown that $\tan 2\theta_{12}$, where θ_{12} is the mixing angle governing the solar neutrino oscillations, is proportional to $\epsilon_{\rm D}\eta_2$, and thus the larger value of $\eta_2 \simeq 10$ is implied. Thus, $\epsilon_{M2} = \epsilon_{\rm D}^4/\eta_2 \simeq 10^{-5}$, i.e., the heaviest Majorana neutrino has a much larger mass than the other two.

We can gain even more insight in the hierarchy of $M_{\rm R}$ by looking at the decay asymmetry of the heavy Majorana neutrinos. It reads

$$\begin{aligned} \varepsilon_1 &= \frac{3m^2}{8\pi v^2}\epsilon_{\rm D}^4 \left(\frac{\epsilon_{M1}}{\epsilon_{M2}}\sin 2\beta + f^2\epsilon_{M1}\sin 2(\beta - \phi)\right) \\ &\simeq 0.1\epsilon_{\rm D}^4 \left(\frac{\epsilon_{M1}}{\epsilon_{M2}}\sin 2\beta + f^2\epsilon_{M1}\sin 2(\beta - \phi)\right) \\ &\simeq 0.1\epsilon_{\rm D}^4\frac{\epsilon_{M1}}{\epsilon_{M2}}\sin 2\beta , \end{aligned} \tag{22}$$

where we used $\epsilon_{M1} \ll 1$ and assumed again $m \simeq v$. We can identify the leptogenesis phase β. Since the decay asymmetry should be negative, we can constrain β to lie between $\pi/2$ and π or between $3\pi/2$ and 2π. In order to reach the favorable value of $|\varepsilon_1| \gtrsim 10^{-7}$, the factor $\epsilon_{M2}/\epsilon_{M1} = \eta_1/\eta_2$ should not exceed ~ 10. Therefore, the two lightest Majorana neutrinos display a rather mild hierarchy. The requirements for the structure of m_ν and successful leptogenesis therefore determine the hierarchy of $M_{\rm R}$.

For numerical estimates of the obtained quantities we shall use in the following the representative values $\epsilon_{M1} = 10^{-6}$, $\epsilon_{M2} = 10^{-5}$ and $\epsilon_D = 0.1$. These choices basically eliminate the parameter $F = f\mathrm{e}^{\mathrm{i}\phi}$ from the problem. The ratios of the branching ratios of the LFV violating charged lepton decays in (27) remain however somewhat sensitive to this parameter. Looking with the given parameter set for $\epsilon_{\rm D}$, ϵ_{M1} and ϵ_{M2} at (21), one notes that the terms including α are subleading. One can therefore expect the phase β to play the major role in the observables under study. We shall see that this is indeed the case.

For thermal leptogenesis the important effective mass parameter is given by

$$\tilde{m}_1 = \frac{(m_{\rm D}^\dagger m_{\rm D})_{11}}{M_1} \simeq \frac{m^2}{M} b^2 \eta_1 \epsilon_{\rm D}^2 , \tag{23}$$

being of the order of the entries in m_ν and thereby guaranteeing for the baryon asymmetry a not too strong wash-out factor κ (stemming from lepton number violating scattering processes) of $\kappa \sim 0.1$–10^{-3} [11].

We can get a lower limit on the heavy neutrino masses by comparing our formula for ε_1 with its analytical upper limit, which reads [15]

$$|\varepsilon_1| \lesssim \frac{3}{8\pi v^2} M_1 \sqrt{\Delta m_{\rm A}^2} . \tag{24}$$

With $\Delta m_{\rm A}^2 \gtrsim 10^{-3}\,\mathrm{eV}^2$ one finds

$$M_1 \gtrsim \epsilon_{\rm D}^4 \frac{\epsilon_{M1}}{\epsilon_{M2}} 10^{15}\,\mathrm{GeV} . \tag{25}$$

Therefore, for our chosen parameters of $\epsilon_{\rm D} \simeq 0.1$ and $\epsilon_{M1}/\epsilon_{M2} \simeq 0.1$, we have $M_1 \gtrsim 10^{10}\,\mathrm{GeV}$.

We can now take a closer look at the rates of the LFV violating charged lepton decays. The assumption of universality of the slepton mass matrices at the GUT scale leads via radiative corrections to non-diagonal entries at low scale, which give rise to LFV violating charged lepton decays such as $\mu \to e + \gamma$, $\tau \to \mu + \gamma$ and $\tau \to e + \gamma$ [16]. The branching ratios for the decay $\ell_j \to \ell_i \gamma$ with $\ell_{(3,2,1)} = \tau, \mu, e$ are approximately proportional to $|(m_{\rm D} m_{\rm D}^\dagger)_{ji}|^2$. In our case, their magnitudes are governed by

$$\mathrm{BR}(\mu \to e\gamma) \propto \left|\left(m_{\rm D} m_{\rm D}^\dagger\right)_{21}\right|^2 \simeq a^4 m^4 \epsilon_{\rm D}^{10}, \tag{26}$$

and their ratios are predicted to be

$$\mathrm{BR}(\mu \to e\gamma) \simeq \frac{1}{g^2}\,\mathrm{BR}(\tau \to e\gamma) \simeq \frac{a^2}{f^2}\epsilon_{\rm D}^6\,\mathrm{BR}(\tau \to \mu\gamma) . \tag{27}$$

This relation gets modified by the presence of small entries in $m_{\rm D}$; see Sect. 4.3.

4.1 Diagonalization

As we have seen, our simple arguments lead to the situation in which one of the right-handed Majorana masses is much heavier than the other two, which in turn display a mild hierarchy. In order to compare our framework with the neutrino data, we shall next diagonalize the resulting mass matrix m_ν, leaving the definitions and details to the appendix. We did not consider the renormalization of the mass matrix since the corrections to neutrino masses and mixings are subleading in the case of a hierarchical mass spectrum [17], which we are considering.

Observation requires a large mixing in the 23 sector of the matrix m_ν in (21), which is given by

$$m_\nu^{23} \simeq \frac{-m^2}{M} \begin{pmatrix} \eta_2 + B^2 \epsilon_{\mathrm{D}}^2 \eta_1 & g\eta_2 \\ \cdot & 1 + g^2 \eta_2 \end{pmatrix} \simeq \frac{-m^2}{M} \eta_2 \begin{pmatrix} 1 & g \\ \cdot & g^2 \end{pmatrix}, \tag{28}$$

and which is diagonalized by the mixing angle

$$\tan 2\theta_{23} \simeq \frac{2g}{g^2 - 1} \,. \tag{29}$$

Note that the hierarchy chosen in this analysis renders the 23 submatrix quasi-real, thereby simplifying the diagonalization procedure; see the appendix for details. In order to guarantee a large solar mixing, the determinant of m_ν^{23} should be small [4, 5], which leads from (28) to $|1 + b^2 g^2 \epsilon_{\mathrm{D}}^2 \eta_1 \mathrm{e}^{2\mathrm{i}\beta}| \lesssim 1$.

The deviation from maximal mixing is of order

$$1 - \sin^2 2\theta_{23} \simeq \left(\frac{1 - g^2}{1 + g^2} \right)^2 \,. \tag{30}$$

The largest eigenvalue of m_ν^{23} is

$$m_3' \simeq \frac{-m^2}{M} \eta_2 \left(1 + g^2\right) \,. \tag{31}$$

Note that m_3' will not be changed significantly by the following two rotations, $m_3' \simeq m_3$, and can therefore already be confronted with $\sqrt{\Delta m_{\mathrm{A}}^2} \simeq 0.05\,\mathrm{eV}$. Values of $m \simeq v$ and $M \simeq 10^{16}\,\mathrm{GeV}$ lead to the desired value if $g \simeq 1$ and $\eta_2 \simeq 10$.

It is now straightforward to extend the diagonalization procedure from Sect. 3 in order to obtain the remaining mass and mixing parameters; see the appendix for details. One finds for the angle θ_{13} that

$$\tan 2\theta_{13} \simeq \sqrt{2} a \epsilon_{\mathrm{D}} \frac{1 + g}{1 + g^2} \,, \tag{32}$$

while the solar neutrino oscillations are triggered by

$$\tan 2\theta_{12} \simeq \frac{\sqrt{2} a \epsilon_{\mathrm{D}} \eta_2 \left(1 - g\right) \left(1 + g^2\right)}{\sqrt{1 + b^2 g^2 \epsilon_{\mathrm{D}}^2 \eta_1 \left(b^2 g^2 \epsilon_{\mathrm{D}}^2 \eta_1 + 2c_{2\beta}\right)}} \,. \tag{33}$$

One notes that θ_{13} is naturally small, $\tan 2\theta_{13} \propto \epsilon_{\mathrm{D}}$, while $\tan 2\theta_{12}$ is larger than $\tan 2\theta_{13}$ by approximately a factor of $\sim \eta_2$. We therefore observe a hierarchy in the mixing angles of the form

$$\tan 2\theta_{23} \propto \frac{1}{1 - g^2} > \tan 2\theta_{12} \propto \epsilon_{\mathrm{D}} \eta_2 > \tan 2\theta_{13} \propto \epsilon_{\mathrm{D}} \,, \tag{34}$$

which is exactly the situation implied by neutrino phenomenology. It is seen that, for $\epsilon_{\mathrm{D}} \simeq 0.1$, a value $\eta_2 \sim 10$ is required in order to reproduce the large solar neutrino mixing angle, which justifies our choice for η_2. Note that the dominator in (33) should be smaller than 1. In fact, the denominator can be identified with $|1 + b^2 g^2 \epsilon_{\mathrm{D}}^2 \eta_1 \mathrm{e}^{2\mathrm{i}\beta}|$, and the condition that this quantity is smaller than 1 was exactly the condition to make the determinant of the 23 submatrix of m_ν small. With our assumptions about the hierarchy parameters we can make the denominator very small for $b \simeq 1$ and $\beta \simeq \pi/2$. This value of β, however, leads via (22) to a too small baryon asymmetry. We have therefore an interplay between the baryon asymmetry of the universe and the non-maximality of θ_{12}, which resembles the situation mentioned for the 2×2 case and discussed at the end of Sect. 3.

Regarding θ_{13}, useful estimates can be performed. First of all, one can expect θ_{13} to be non-zero, because $a = 0$ will lead to a too small solar neutrino mixing. More precisely, we have for $g \simeq 1$ the estimate

$$|U_{e3}|^2 \simeq \frac{a^2 \epsilon_{\mathrm{D}}^2}{2} \sim \left(10^{-3}\text{–}10^{-2}\right) \,, \tag{35}$$

where we assumed a to be between 0.5 and 3 and $\epsilon_{\mathrm{D}} = 0.1$. These values can be tested in the not so far future [18]. The magnitude of U_{e3} is decisive for many neutrino mass models [19].

Figure 2 shows for $\epsilon_{\mathrm{D}} = 0.1$, $\epsilon_{M1} = 10^{-6}$ and $\epsilon_{M2} = 10^{-5}$ the mixing parameter $\tan^2 \theta_{12}$ as obtained from (33) for specific choices of a, b and g as a function of the leptogenesis phase β. The values of θ_{23} are close to maximal and the ones of $\sin^2 \theta_{13}$ close to 10^{-2} for all cases plotted, confirming our quantitative statements from above. Also shown is – when negative – the decay asymmetry ε_1 from (22) multiplied with -10^5. Its value is of the required magnitude for the solar neutrino mixing angle inside its experimental range, the angle θ_{13} below its upper limit and atmospheric mixing very large. Note that too large $\tan^2 \theta_{12}$ can lead to a too small decay asymmetry.

The two remaining mass eigenvalues are complicated functions of the parameters η_1, η_2, ϵ_{D}, a, b, g, α and β. We saw above that for $\eta_2 \simeq 10$ and $M \simeq 10^{16}\,\mathrm{GeV}$ the favorable value of $m_3 \simeq \sqrt{\Delta m_{\mathrm{A}}^2}$ is achieved. With this choice for M, the common factor of $m_{1,2}$ is $m^2/M \simeq 3 \cdot 10^{-3}\,\mathrm{eV}$, which, when multiplied with a sum and difference of two terms of order 1, can, admittedly involving some tuning, result in the required values of $|m_2|^2 - |m_1|^2 = \Delta m_\odot^2$. For later use we define $\Delta m_\odot^2 = m^4/M^2 \tilde{s}$, where $\tilde{s}$ is a function of the hierarchy parameters ϵ_{D}, $\eta_{1,2}$ and the mass matrix parameters a, b, g, α and β. Its value is for $m \simeq v$ and $M \simeq 10^{16}\,\mathrm{GeV}$ located around 10.

4.2 *CP* violation in neutrino oscillation experiments and neutrinoless double beta decay

We shall investigate now the predictions of the scenario for the *CP* asymmetries in neutrino oscillation experiments and for neutrinoless double beta decay and its connection to leptogenesis. The interplay between these low and high energy parameters has recently been analyzed in a number

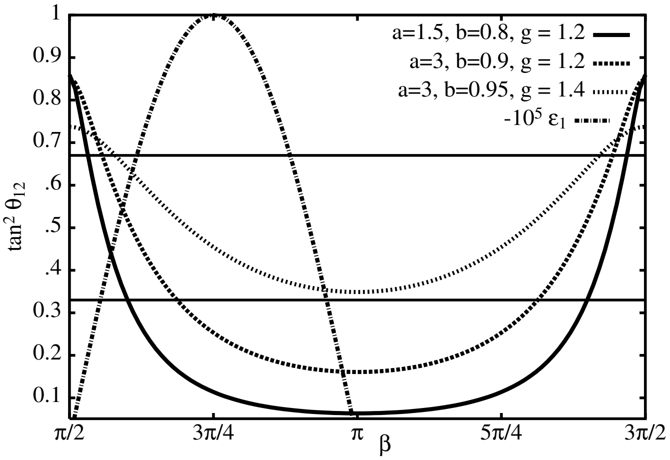

Fig. 2. Result for the mixing parameter $\tan^2\theta_{12}$, as obtained from (33), for different a, b and g as a function of the leptogenesis phase β. The range as implied by experiment is indicated. The values of $|U_{e3}|^2$ are 0.009, 0.033 and 0.027, respectively. For $g = 1.2$ (1.4) atmospheric neutrino mixing is given by $\sin^2 2\theta_{23} \simeq 0.97$ (0.90). Plotted is also the decay asymmetry ε_1 from (22) multiplied with -10^5 (dash-dotted)

of publications [14,20,22]. Instead of trying to identify the low energy Dirac and Majorana phases and express them in terms of the available high energy phases in (19), we shall work as convention independent as possible.

We can calculate the rephasing invariant CP observable J_{CP}, which can be written as [22]

$$J_{CP} = -\frac{\mathrm{Im}\,(h_{12}h_{23}h_{31})}{\Delta m^2_{21}\Delta m^2_{31}\Delta m^2_{32}}\,,\ \text{where } h = m_\nu m_\nu^\dagger\,. \quad (36)$$

With the help of m_ν given in (21) we find with the choice of $\epsilon_{\mathrm{D}}^2\eta_1 \simeq 1$ and $\eta_1 \simeq 10\eta_2$ that the leading term is given by

$$\begin{aligned} -\mathrm{Im}\,(h_{12}h_{23}h_{31}) &\simeq \frac{m^{12}}{M^6}\epsilon_{\mathrm{D}}^4\eta_1\eta_2^4a^2b^2g^2\left(1+g^2\right)\sin 2\beta \\ &\simeq \frac{2m^{12}}{M^6}\epsilon_{\mathrm{D}}^4\eta_1\eta_2^4a^2b^2\sin 2\beta\,. \end{aligned} \quad (37)$$

With the help of $\Delta m^2_{31} \simeq \Delta m^2_{32} \simeq m_3^2 \simeq \left(2\eta_2 m^2/M\right)^2$ we find with our definition for $\Delta m^2_\odot$ that in leading order

$$J_{CP} \simeq \frac{1}{8}\epsilon_{\mathrm{D}}^4\eta_1a^2b^2\tilde{s}\sin 2\beta\,. \quad (38)$$

For our representative values we find that $J_{CP} \sim 10^{-2}a^2b^2 \times \sin 2\beta$. Recall that for, e.g., $\tan^2\theta_{12} = 0.45$, $\sin^2 2\theta_{23} = 1$ and $\sin^2\theta_{13} = 0.01$ the invariant J_{CP} is given by

$$\begin{aligned} J_{CP} &= \mathrm{Im}\left\{U_{e1}U_{\mu 1}^*U_{e2}^*U_{\mu 2}\right\} \\ &= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta \\ &\simeq 0.02\sin\delta\,. \end{aligned} \quad (39)$$

Thus, it is confirmed that θ_{13} is sizable in the framework under study. Since $\Delta m^2_\odot = |m_2|^2 - |m_1|^2$ depends on η_1, η_2, ϵ_{D}, a, b, g, α and β, whereas the decay asymmetry is proportional to $\sin 2\beta$, there is no simple connection between the size of J_{CP} and Y_B. It is seen, however, that – due to the same dependence on β – vanishing J_{CP} is incompatible with successful leptogenesis and that J_{CP} has the same sign as the baryon asymmetry. The case $\epsilon_{\mathrm{D}} = 0$, i.e., the presence of only one Dirac mass, corresponds to an effective two flavor system in which J_{CP} has to vanish, as confirmed by (38).

Finally we can analyze the prediction of the scenario for neutrinoless double beta decay. From (21) and our usual assumptions of the parameters we find that the absolute values of the ee element of m_ν is

$$\langle m\rangle \equiv |(m_\nu)_{ee}| = \frac{m^2}{M}a^2\epsilon_{\mathrm{D}}^2\eta_2 \simeq 3a\cdot 10^{-4}\mathrm{eV}\,. \quad (40)$$

Neutrinoless double beta decay triggered by values of $\langle m\rangle$ smaller than 10^{-3} eV will be unobservable [23]. With Eqs. (31) and (35) we can however write an interesting correlation of parameters, namely:

$$\langle m\rangle \simeq \sqrt{\Delta m^2_{\mathrm{A}}}\,|U_{e3}|^2\,. \quad (41)$$

In summary, the same phase governs the CP asymmetry in neutrino oscillations and the decay asymmetry, whereas there is no correlation of the leptogenesis phase with the effective mass in neutrinoless double beta decay. The very same features have been found for the minimal see-saw model [21], which is defined as having only 2 heavy Majorana neutrinos and 2 zeros in the Dirac mass matrix.

Given the presence of two zeros (or very small entries) in our m_D (see Eq. (19)) and the fact that $M_3 \gg M_{2,1}$, it is very interesting that we encounter the same situation.

4.3 Effects of entries of order ϵ_{D}^4 in m_{D}

The question arises if it is valid to neglect terms of order ϵ_{D}^4 in the 11, 13 and 31 entries of m_{D} in (19). We therefore repeat the calculation with terms of this order. One finds that new contributions to m_ν are suppressed by one or two orders of ϵ_{D}. Regarding the LFV violating decays, one observes that the term $|(m_{\mathrm{D}}m_{\mathrm{D}}^\dagger)_{31}|^2$ now has the leading contribution proportional to $\epsilon_{\mathrm{D}}^4 h_1$, where h_1 is the absolute value of the 13 element of m_{D}. The other terms acquire subleading new contributions stemming from the new entries in m_{D}. Thus, (27) is modified to

$$\begin{aligned} \mathrm{BR}(\mu\to e\gamma) &\simeq \frac{a^2}{h_1^2}\epsilon_{\mathrm{D}}^2\mathrm{BR}(\tau\to e\gamma) \\ &\simeq \frac{a^2}{f^2}\epsilon_{\mathrm{D}}^6\,\mathrm{BR}(\tau\to\mu\gamma)\,, \end{aligned} \quad (42)$$

or, numerically,

$$\mathrm{BR}(\mu\to e\gamma) \sim 10^{-2}\,\mathrm{BR}(\tau\to e\gamma)$$

$$\sim 10^{-6}\,\mathrm{BR}(\tau \to \mu\gamma)\,. \qquad (43)$$

Note the analogy of these ratios with the ones presented in [14], where also a hierarchical m_{D} was assumed. One sees that the small entries of order ϵ_{D}^4 change the ratio between $\mathrm{BR}(\mu \to e\gamma)$ and $\mathrm{BR}(\tau \to e\gamma)$ by a factor of $\epsilon_{\mathrm{D}}^2 \simeq 10^{-2}$.

The decay asymmetry ε_1 is also slightly altered. It reads now

$$\varepsilon_1 = \frac{3m^2}{8\pi v^2}\epsilon_{\mathrm{D}}^2\left(\epsilon_{\mathrm{D}}^2\frac{\epsilon_{M1}}{\epsilon_{M2}}\sin 2\beta + \frac{h_2^2}{b^2}\epsilon_{M1}\sin 2\delta_2\right)\,, \qquad (44)$$

where h_2 and δ_2 are the absolute value and phase of the 31 entry of m_{D}. For $\epsilon_{M1} \ll \epsilon_{\mathrm{D}}^2$, the situation we are interested in, we recover the form given in (22). Thus, small entries in m_{D}, which were neglected in (19), have in our framework some influence on the ratios of the LFV violating decay branching ratios but only little influence on m_ν and ε_1.

5 Conclusions

The see-saw mechanism with hierarchical Dirac and Majorana neutrino masses was reanalyzed in the presence of CP phases. A consistent and appealing framework of neutrino mixing phenomenology and leptogenesis was found, in which one of the heavy Majorana neutrinos is much heavier than the other two, which in turn display a mild hierarchy. It was investigated how large neutrino mixing can be generated starting from hierarchical mass matrices in the see-saw mechanism.

Ratios for the branching ratios of LFV charged lepton decays are predicted, which are sensitive to small entries in m_{D}. A natural hierarchy of the mixing angles in accordance with observation is found and it holds $|U_{e3}|^2 \gtrsim 10^{-3}$, which is observable in the not so far future. There can be an interplay between too large solar neutrino mixing and a too small baryon asymmetry.

The CP asymmetry in neutrino oscillations has the same sign as the baryon asymmetry of the universe and successful leptogenesis implies non-zero and measurable J_{CP}. Neutrinoless double beta is not linked with the leptogenesis phase and will probably not be observable. The framework under study resembles in this respect very much the minimal see-saw model.

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A Diagonalization of a complex and hierarchical symmetric 3×3 matrix

We present for completeness our formulae for the diagonalization of a complex and hierarchical symmetric 3×3 matrix. It is a special case of the general strategy as outlined, e.g., in [5]. In the diagonalization of a 2×2 matrix three phases were present. We saw that two of them can be absorbed in the charged lepton fields. Diagonalizing a complex 3×3 matrix through three consecutive 2×2 diagonalizations will introduce six phases, which in principle can influence the mixing angles. In our case, however, they do not. We take advantage of the somewhat more simple structure of m_ν in the hierarchical situation we consider. It is convenient to express the results in terms of mixing angles. Regarding the phases, as stated in the text, we prefer not to identify the low energy Dirac and Majorana phases but work with convention independent quantities like J_{CP}. Consider a symmetrical neutrino mass matrix

$$m = \begin{pmatrix} a & b & d \\ \cdot & e & f \\ \cdot & \cdot & g \end{pmatrix}\,, \qquad (45)$$

where the 23 block has entries larger than the other elements. The strategy outlined in [5] is to first rephase the mass matrix with $P_2 m P_2$, where P_2 is a diagonal phase matrix with complex entries on the 22 and 33 elements. Then, one puts zeros in the 23 and 13 elements of m by diagonalizing first the 23 submatrix and then the resulting 13 submatrix. Then the matrix is again rephased by a diagonal phase matrix containing only one complex entry on the 22 element. After that, we have to diagonalize the 12 submatrix and end up in this way with a diagonal matrix. The eigenstates are however still complex. Thus, by again rephasing the diagonal matrix and absorbing these three phases in the charged leptons, we end up with the desired three real diagonal entries, three mixing angels and three phases.

In our case, the 23 submatrix of (21) is effectively real, since we choose $\eta_2 \simeq 10$. Therefore, the first rephasing with P_2 is not necessary and there is also no phase in the 23 rotation. Thus, the 23 submatrix is diagonalized via $R_{23}^{\mathrm{T}} m R_{23}$ where

$$R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}\,, \qquad (46)$$

where $c_{23} = \cos\theta_{23}$ and $s_{23} = \sin\theta_{23}$. The resulting matrix m' is

$$m' = \begin{pmatrix} a & bc_{23} - ds_{23} & bs_{23} + dc_{23} \\ \cdot & m_2' & 0 \\ \cdot & \cdot & m_3' \end{pmatrix} \equiv \begin{pmatrix} a & b' & d' \\ \cdot & m_2' & 0 \\ \cdot & \cdot & m_3' \end{pmatrix}\,, \qquad (47)$$

for

$$m_{2,3}' = \frac{1}{2}\left((e+g) \mp \sqrt{(e-g)^2 + 4f^2}\right) \qquad (48)$$

and

$$\tan 2\theta_{23} = \frac{2f}{g-e}\,. \qquad (49)$$

Now the 13 submatrix of m' is diagonalized via $R_{13}^{\mathrm{T}} m' R_{13}$ with

$$R_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13}^* & 0 & c_{13} \end{pmatrix} , \tag{50}$$

where $c_{13} = \cos\theta_{13}$ and $s_{13} = \sin\theta_{13} \mathrm{e}^{\mathrm{i}\phi_{13}}$. The resulting matrix m'' reads

$$m'' = \begin{pmatrix} m_1'' & b' c_{13} & 0 \\ \cdot & m_2' & b' s_{13} \\ \cdot & \cdot & m_3'' \end{pmatrix} \simeq \begin{pmatrix} m_1'' & b' & 0 \\ \cdot & m_2' & 0 \\ \cdot & \cdot & m_3'' \end{pmatrix} , \tag{51}$$

where the last approximation takes into account the smallness of θ_{13} as implied by the reactor experiments and the hierarchical structure of m. The masses and the mixing angle are given by

$$m_{1,3}'' = \frac{1}{2} \left((a + m_3') \mp \sqrt{(a - m_3')^2 + 4 d'^2} \right) \tag{52}$$

and

$$\tan 2\theta_{13} = \frac{2d'}{m_3' \mathrm{e}^{-\mathrm{i}\phi_{13}} - a \mathrm{e}^{\mathrm{i}\phi_{13}}} \simeq \frac{2d' \mathrm{e}^{\mathrm{i}\phi_{13}}}{m_3'} ,$$

where

$$\begin{aligned} \arg(d') &= \arg\left(m_3' \mathrm{e}^{-\mathrm{i}\phi_{13}} - a \mathrm{e}^{\mathrm{i}\phi_{13}}\right) \\ &\Rightarrow \phi_{13} \simeq \arg(m_3') - \arg(d') . \end{aligned} \tag{53}$$

From (21) we see that the 11 element of our m_ν (here called a) is much smaller than m_3' as given in (31). The phase ϕ_{13} is therefore suppressed and does not influence the magnitude of θ_{13}. The eigenvalue $m_3'' \equiv m_3$ is already the heaviest eigenvalue of the matrix m. Now we rephase m'' through a diagonal phase matrix P with only the 22 entry being complex, $P = \mathrm{diag}\left(1, \mathrm{e}^{\mathrm{i}\phi}, 1\right)$. Finally, the 12 submatrix of m'' gets diagonalized by $R_{12}^{\mathrm{T}} m'' R_{12}$ where

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}^* & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{54}$$

and for the masses and mixing angle

$$m_{1,2} = \frac{1}{2} \left((m_1'' + m_2') \mp \sqrt{(m_1'' - m_2')^2 + 4 b'^2} \right) \tag{55}$$

as well as

$$\tan 2\theta_{12} = \frac{2b' \mathrm{e}^{\mathrm{i}\phi}}{m_2' \mathrm{e}^{-\mathrm{i}\phi_{12}} \mathrm{e}^{2\mathrm{i}\phi} - m_1'' \mathrm{e}^{\mathrm{i}\phi_{12}}} ,$$

where

$$\arg\left(b' \mathrm{e}^{\mathrm{i}\phi}\right) = \arg\left(m_2' \mathrm{e}^{-\mathrm{i}\phi_{12}} \mathrm{e}^{2\mathrm{i}\phi} - m_1'' \mathrm{e}^{\mathrm{i}\phi_{12}}\right) . \tag{56}$$

In our case it turns out that $m_2' \gg m_1''$; therefore, ϕ and ϕ_{12} do not influence the magnitude of θ_{12}. The mass states are in general still complex. Rephasing these states through a diagonal phase matrix and absorbing them in the charged lepton fields then leaves us with the correct number of three phases in U.

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